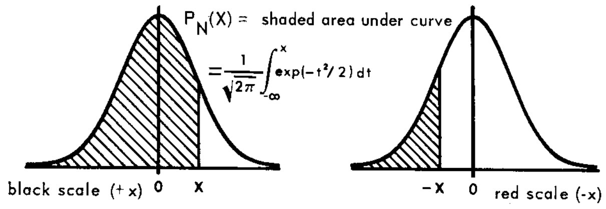
PROBABILITY SLIDE RULE INSTRUCTIONS

This slide rule is a highly accurate device for quick and easy numerical evaluation of several important probability functions. As such, it can serve the student of probability or statistics as well as the practicing engineer, scientist, or mathematician. The instructions presented below illustrate the features of this slide rule and point out some application areas.

The slide rule consists of two independent sides. Side one, labeled "Probability Slide Rule," permits direct reading of probability values for three probability distributions: Normal, Rayleigh, and Maxwell. Side two, labeled "Multiple Event probability," uses a rotatable inner wheel to determine the probability of either zero successes or at least one success in N independent trials. Both sides of the slide rule eliminate the necessity for looking up numerical values in mathematical tables, with significant savings in time. The reading of inverse results is particulary simple.

1. Normal Distribution Scale

This scale is used to find probability values, or inversely, percentiles on the standardized (zero mean, unit standard deviation) normal distribution. The normal distribution is also known as the Gaussian distribution, and the one-dimensional error distribution. The sketch below indicates the value of $P_N(x)$ on the normal scale which corresponds to a given x on the outer scale.


$P_N(X)$ = shaded area under curve
$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-t^2/2)\, dt$
black scale (+x) 0 X
-X 0 red scale (-x)

Normal Distribution Scales
($\mu_N = 0$, $\sigma_N = 1.0$)

The normal distribution function is defined for both positive and negative values of x. The black scale is used for positive x. The red scale, which is obtained by the relation $P_N(\text{red}) = 1 - P_N(\text{black})$, is used for negative x. In order to standardize variables, if y represents an original random variable with mean μ and standard deviation σ, then the transformation $x = (y - \mu)/\sigma$ must be performed. That is, one must convert to a quantity which gives the number of standard deviations to the right or left of the mean.

The normal distribution arises in many physical situations involving one-dimensional random data. For example, psychologists commonly assume that the scores of individuals on various intelligence tests are normally distributed. The amplitude time history of the response of structures to acoustical excitation is assumed to be normal in many cases. Electrical "noise" occurring in many electronic circuits as measured by voltage fluctuations is approximately normal. Physical characteristics such as heights of human beings may be approximated by a normal distribution.

The following problems illustrate use of the normal scale.

(a) Find the probability that x is less than or equal to 1.3. (In equation form, $\text{Prob}\left[x \leq 1.3 \right] = ?$) Place the hairline above 1.3 on the outer scale, then read the value $P_N(1.3) = .903$ below the hairline on the black normal scale.

(b) Find the probability that x is less than or equal to minus 0.8 ($\text{Prob}\left[x \leq -0.8 \right] = ?$). Since x is negative, the red scale must be used. Place the hairline above 0.8 on the outer scale, then read the value $P_N(-0.8) = .212$ below the hairline on the red normal scale.

(c) Find the probability that x is between minus 1.1 and plus 2.2 ($\text{Prob}\left[-1.1 < x \leq 2.2 \right] = ?$). First place the hairline above 2.2 on the outer scale and read the value $P_N(2.2) = .986$ below the hairline on the black normal scale. Now one must subtract the probability of being less than -1.1 in order to obtain the probability of being between the two limits -1.1 and 2.2. Hence, place the hairline above 1.1 on the outer scale and read the value $P_N(-1.1) = .136$ below the hairline on the red normal scale. Then,

$$\text{Prob}\left[-1.1 < x \leq 2.2 \right] = P_N(2.2) - P_N(-1.1) = .986 - .136 = .850$$

(d) Find the 80th percentile of the normal distribution. That is, find that value of x denoted by $x_{.80}$ such that $P_N(x_{.80}) = \text{Prob}\left[-\infty < x \leq x_{.80} \right] = .80$. This is an inverse problem. To solve this problem, place the hairline above $P_N = .80$ on the black normal scale and read $x_{.80} = .841$ on the outer scale.

(e) Find the value of x such that the interval between minus x and plus x contains 95% of the normal distribution. This is an inverse problem. By symmetry, one desires here $P_N(x) - P_N(-x) = 0.975 - 0.025 = 0.95$. Hence, one finds either the 97.5 percentile or the negative of the 2.5 percentile. Place the hairline above .975 on the black normal scale, then read $x = 1.96$ below the hairline on the outer scale.

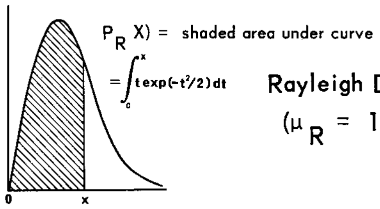
(f) Find the probability that y is less than or equal to 66.5 when the mean value $\mu = 60$ and the standard deviation $\sigma = 5.0$. Here, it is necessary to standardize the variable by using the transformation $x = (y - \mu)/\sigma$. For these values, $x = (66.5 - 60)/5.0 = 1.3$. The desired answer is now given by $P_N(1.3) = .903$ using the procedure given in Example (a).

(g) Find the 80th percentile of the normal distribution for y when $\mu = 60$ and $\sigma = 5.0$. This is an inverse problem. First, find the 80th percentile of the standardized normal distribution. As in Example (d), $x_{.80} = .841$. Now the transformation $x = (y - \mu)/\sigma$ should be solved for y to give the desired answer

$$y_{.80} = \mu + \sigma x_{.80} = 60 + 5.0(.841) = 64.2.$$

2. Rayleigh Distribution Scale

This scale is used to find probability values, or inversely, percentiles on the Rayleigh distribution. The Rayleigh distribution is also known as the radial normal distribution, and the two-dimensional error distribution. This distribution is defined only for positive x, and is zero for negative x. The sketch below indicates the value of $P_R(x)$ on the Rayleigh scale which corresponds to a given x on the outer scale.



Rayleigh Distribution Scale
($\mu_R = 1.25$, $\sigma_R = 0.43$)

The Rayleigh distribution occurs in physical situations involving two-dimensional random data. This has application to such problems as determining missed target probabilities in two dimensions, and the probability of peak values of narrow band random processes. The distance $x = \sqrt{t_1^2 + t_2^2}$ from the center of a target is a Rayleigh variable if t_1 and t_2 are two target coordinates which are independent and normally distributed. The following problems illustrate use of the Rayleigh scale.

(a) Find the probability that x is less than 2.5. Place the hairline above 2.5 on the outer scale and read $P_R(2.5) = .956$ on the Rayleigh scale.

(b) Find the probability that x lies between 0.5 and 1.5. Here,

$$\text{Prob}\left[0.5 < x \le 1.5\right] = \text{Prob}\left[x \le 1.5\right] - \text{Prob}\left[x \le 0.5\right]$$
$$= P_R(1.5) - P_R(0.5) = .655 - .117 = .538$$

(c) Find the 95th percentile of the Rayleigh distribution. Proceed exactly as in example (d) for The Normal Distribution.

3. Maxwell Distribution Scale

This scale is used to find probability values, or inversely, percentiles on the Maxwell distribution. The Maxwell distribution is also known as the three-dimensional error distribution, and is a direct extension of the missed target concept to three dimensions. The distance $x = \sqrt{t_1^2 + t_2^2 + t_3^2}$ from the center of a target is a Maxwell variable if t_1, t_2, and t_3 are three target coordinates which are independent and normally distributed.

Features of the Maxwell distribution are identical to that of the Rayleigh distribution; hence, no additional instructions or examples are given here. One may refer directly to the details for the Rayleigh distribution by substituting the word Maxwell for Rayleigh and $P_M(x)$ for $P_R(x)$. For the Maxwell distribution, $\mu_M = 1.60$ and $\sigma_M = 0.45$.

MULTIPLE EVENT PROBABILITY

Two scales are used to find the probability of no occurrence P_0, or at least one occurrence P_1, of a desired event if an experiment (measurement, game) is repeated N times. The probability of occurrence of the desired event in any one experiment is denoted by P. It is assumed that the N experiments are statistically independent, and that P is the same for every experiment. A sequence of N such events is known as "Bernoulli trials." Applications can be made to many fields.

For example, in games of chance, ten flips of an unbiased coin is a sequence of $N = 10$ Bernoulli trials where $P = 0.50$. Three throws of a single fair die, where the event is one particular number, is a sequence of $N = 3$ Bernoulli trials with $P = 1/6$. If a pair of fair dice are involved, the sum of 5 will occur up with probability $P = 1/9$ since the event "sum = 5" can occur in 4 possible ways (1-4, 2-3, 3-2, 4-1) out of a total of 36 possible outcomes. Basic instructions for applying the multiple event probability scales are given on the slide rule itself. The following problems illustrate use of these scales.

(a) Find the probability of obtaining no heads in $N = 10$ tosses of an unbiased coin. First locate $P = .50$ on the black inner movable scale and rotate this scale until it matches with 1.0 on the outer scale. Then place the hairline above $N = 10$ on the outer scale and read the answer $P_0 = .001$ on the red inner scale. In words, the probability of no heads in ten tosses is equal to one in a thousand. Equivalently, the probability of obtaining at least one head in 10 tosses is given on the black inner scale by $P_1 = 1 - P_0 = .999$.

(b) Care must be exercised here not to attempt to read probabilities past this point. That is, the black scale ends at .999 but may be extended as follows: If $N = 20$ is used in the above problem, one would write

$$P_0 = (1 - P)^{20} = (1 - P)^{10}(1 - P)^{10} = (.001)(.001) = 10^{-6}$$

(c) Find the probability of obtaining at least one 5 in 3 throws of an unbiased die. The probability of a 5 on one throw is $P = 1/6 = .167$. Hence, align $P = .167$ on the black movable scale with 1.0 on the outer scale. Place the hairline above $N = 3$ on the outer scale and read the answer $P_1 = .421$ on the black inner scale.

(d) Find the number N of tosses required for a pair of unbiased dice to give a probability of .80 that at least one of the tosses will be a 5. This is an inverse problem. The single event probability is here $P = 1/9 = .111$. First, locate $P = .111$ on the black inner movable scale and rotate this scale until it matches with 1.0 on the outer scale. Then place the hairline over $P_1 = 0.80$ on the black inner scale and read the answer $N = 14$ on the outer scale.

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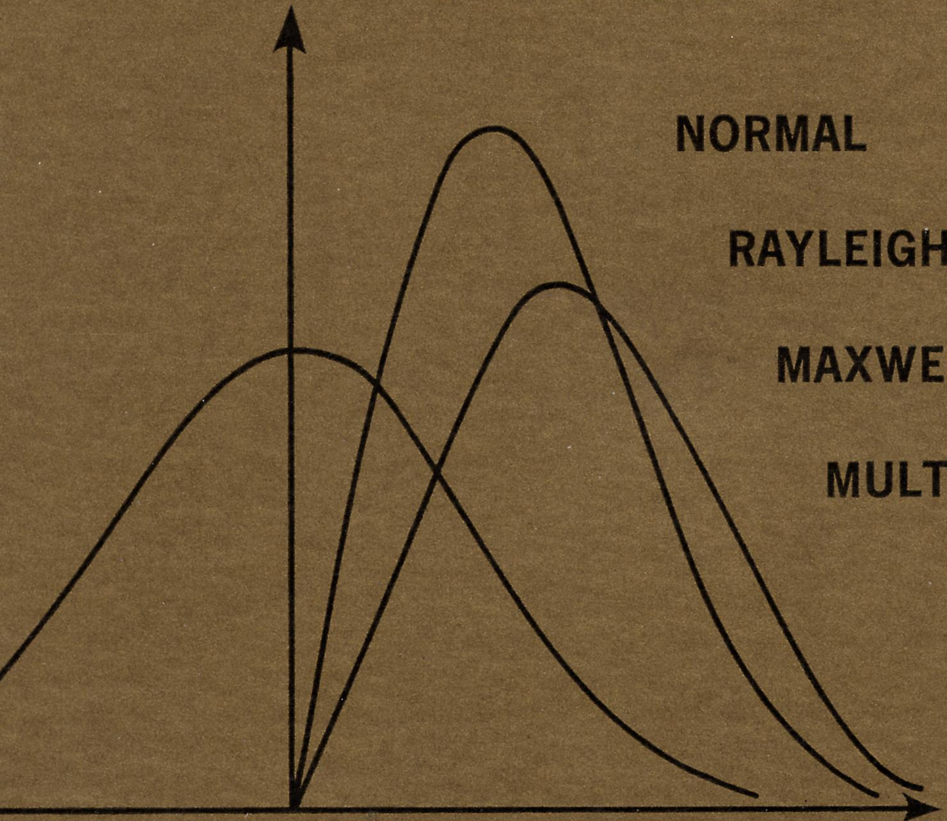
PROBABILITY SLIDE RULE



NORMAL

RAYLEIGH

MAXWELL

MULTIPLE EVENTS

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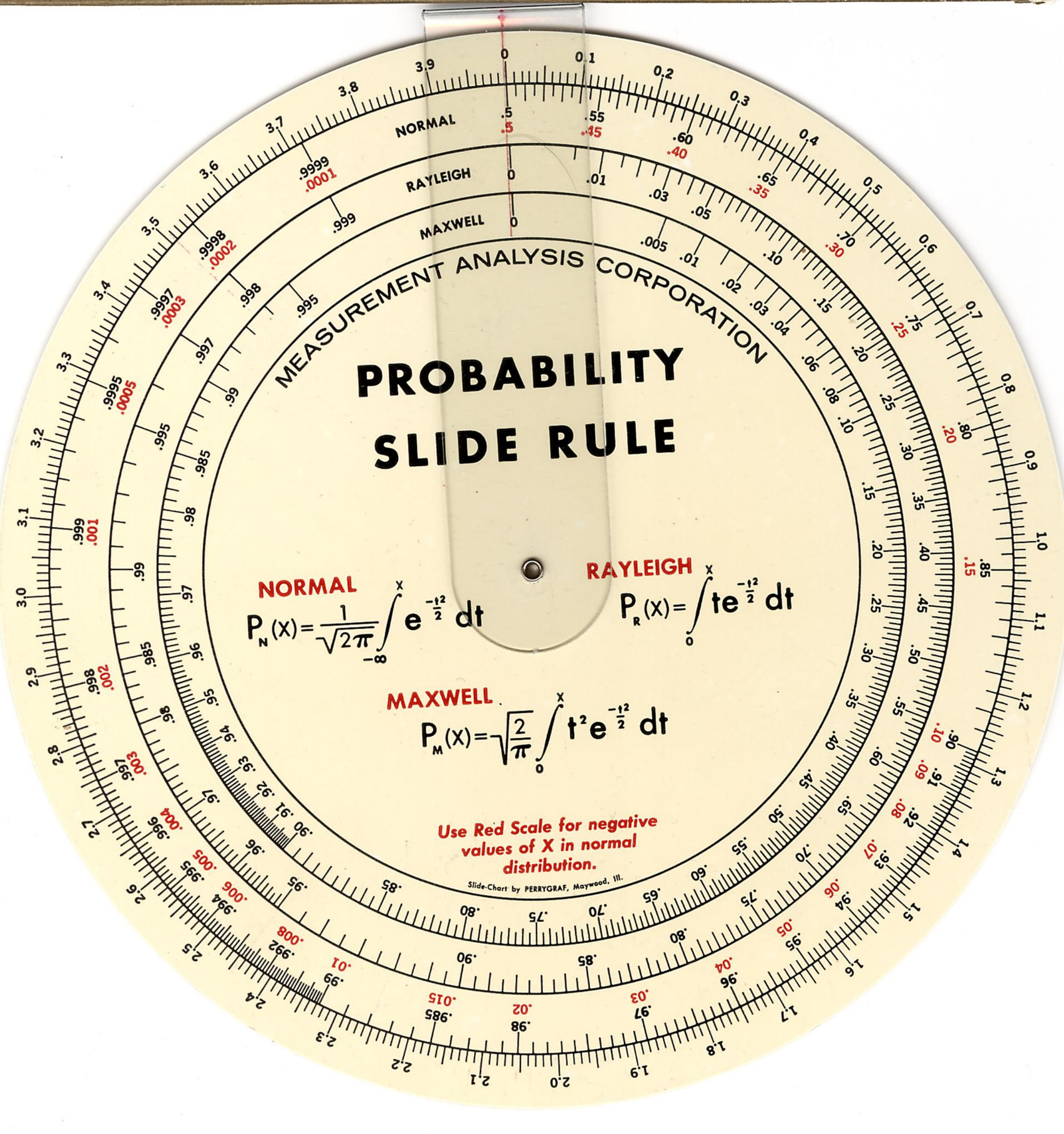

MEASUREMENT ANALYSIS CORPORATION

PROBABILITY SLIDE RULE

NORMAL

$$P_N(X)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{X} e^{-\frac{t^2}{2}}\,dt$$

RAYLEIGH

$$P_R(X)=\int_{0}^{X} t e^{-\frac{t^2}{2}}\,dt$$

MAXWELL

$$P_M(X)=\sqrt{\frac{2}{\pi}}\int_{0}^{X} t^2 e^{-\frac{t^2}{2}}\,dt$$

Use Red Scale for negative values of X in normal distribution.

Slide-Chart by PERRYGRAF, Maywood, Ill.

FOR STUDENTS OF PROBABILITY AND STATISTICS

PRACTICING ENGINEERS, SCIENTISTS, MATHEMATICIANS

Useful For

- **Analysis of Random Data**
- **Design of Experiments**
- **Testing of Hypothesis**
- **Prediction of Results**

APPLICABLE TO MANY FIELDS AND PROBLEMS

Separate scales compute error probability distributions for one, two, and three dimensions.

Multiple Event Probability scales compute probability of no successes, or at least one success, in N independent trials.

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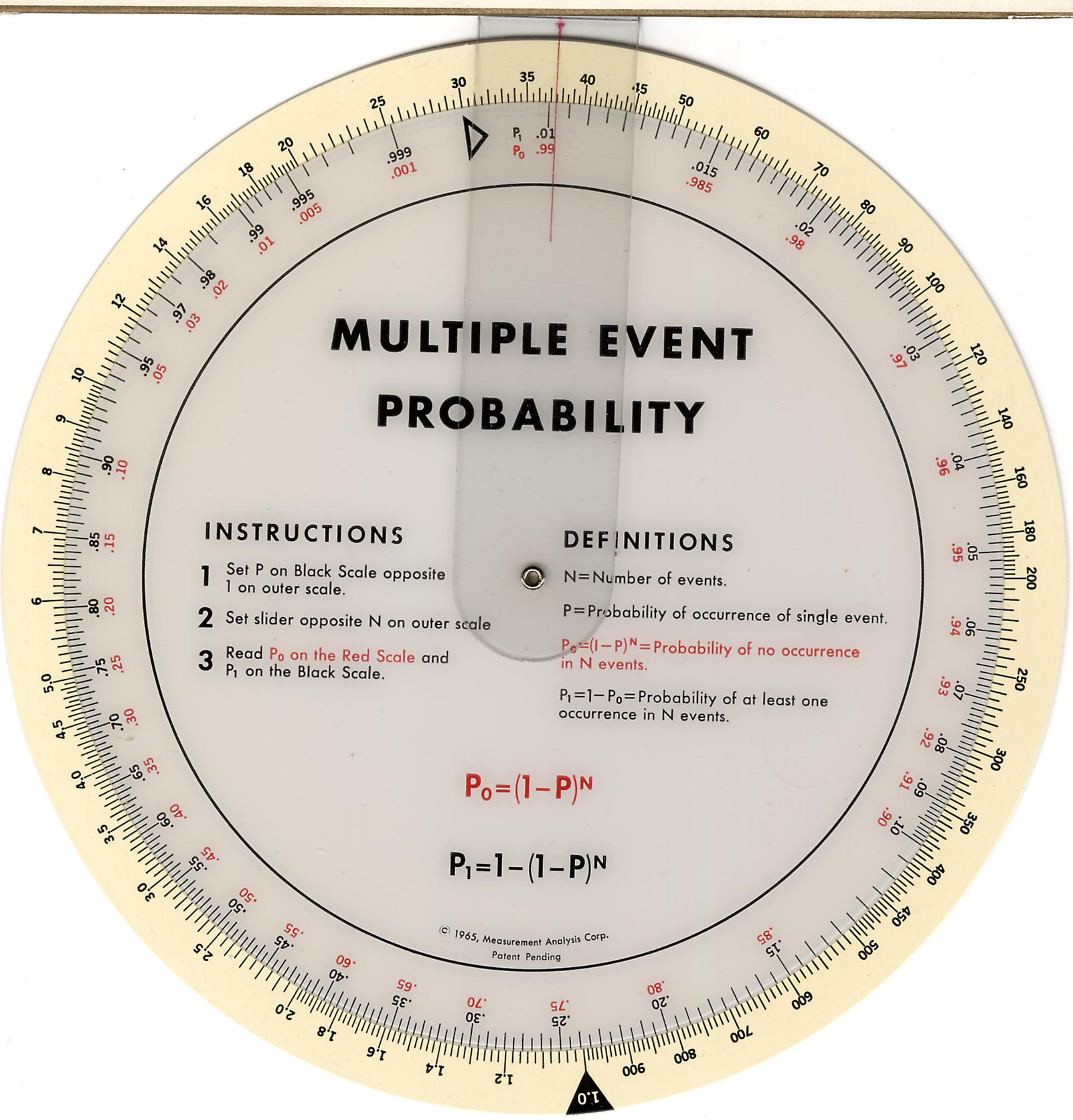

MULTIPLE EVENT PROBABILITY

INSTRUCTIONS

1. Set P on Black Scale opposite 1 on outer scale.
2. Set slider opposite N on outer scale
3. Read P_0 on the Red Scale and P_1 on the Black Scale.

DEFINITIONS

N=Number of events.

P=Probability of occurrence of single event.

$P_0=(1-P)^N$=Probability of no occurrence in N events.

$P_1=1-P_0$=Probability of at least one occurrence in N events.

$$P_0=(1-P)^N$$

$$P_1=1-(1-P)^N$$

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Patent Pending